# Differential Equations

Pure: Calculus · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

**USEFUL EQUATIONS**

$dy/dx = f(x)g(y) \Rightarrow \int dy/g(y) = \int f(x)\,dx$ $dN/dt = kN \Rightarrow N = N_0e^{kt}$ $dT/dt = -k(T - T_s)$

$\ln|y| = F(x) + c \Rightarrow y = Ae^{F(x)}$

## Questions

Show your working in the space under each question.

**1.** EASY Solve $dy/dx = e^{-x}$ with $y = 2$ when $x = 0$.

**2.** EASY Verify that $y = Ae^{3x}$ satisfies $dy/dx = 3y$.

**3.** MEDIUM Solve $dy/dx = x/y$ with $y = 3$ when $x = 0$.

**4.** MEDIUM Solve $dy/dx = y\cos x$ with $y = 1$ when $x = 0$.

**5.** HARD A drink cools in a 20 °C room from 80 °C to 50 °C in 10 minutes. Find its temperature after 20 minutes.

**6.** HARD Solve $dy/dx = (1 + y)/x$ for $x > 0$, given $y = 1$ when $x = 1$.

## Step-by-step answers

Differential Equations

1. $y = -e^{-x} + c$. At $x = 0$: $2 = -1 + c$, so $c = 3$. **$y = 3 - e^{-x}$**.
2. $dy/dx = 3Ae^{3x} = 3y$. **It does, for any A**.
3. $y\,dy = x\,dx$ gives $y^2/2 = x^2/2 + c$. With (0, 3): $y^2 = x^2 + 9$, so **$y = \sqrt{(x^2 + 9)}$**.
4. $dy/y = \cos x\,dx$ gives $\ln y = \sin x + c$, and $c = 0$. **$y = e^{\sin x}$**.
5. $T - 20 = 60e^{-kt}$. At $t = 10$: $30 = 60e^{-10k}$, so $e^{-10k} = 1/2$. At $t = 20$: $T = 20 + 60 \times 1/4 =$ **35 °C**.
6. $dy/(1 + y) = dx/x$ gives $\ln(1 + y) = \ln x + c$, so $1 + y = Ax$. At (1, 1): $A = 2$. **$y = 2x - 1$**.