Further Differentiation

Pure: Calculus · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$(e^{kx})' = ke^{kx}$, $(\ln x)' = 1/x$ $(\sin kx)' = k\cos kx$, $(\cos kx)' = -k\sin kx$

$(uv)' = u'v + uv'$, $(u/v)' = (u'v - uv')/v^2$ $dy/dx = (dy/dt)/(dx/dt)$

Questions

Show your working in the space under each question.

1. EASY Differentiate $y = \cos 4x$.

2. EASY Differentiate $y = e^{x^2}$.

3. MEDIUM Differentiate $y = x/(x^2 + 1)$.

4. MEDIUM Find the stationary point of $y = xe^{-2x}$.

5. HARD Find dy/dx at (1, 2) on the curve $x^2 + 3xy + y^2 = 11$.

6. HARD The radius of a sphere increases at 0.2 cm/s. Find the rate of increase of its volume when r = 5 cm.

Step-by-step answers

Further Differentiation

1. **$dy/dx = -4 \sin 4x$**.
2. Inside x^2 differentiates to $2x$: **$dy/dx = 2xe^{x^2}$**.
3. Quotient rule: $[1(x^2 + 1) - x(2x)]/(x^2 + 1)^2$ = **$(1 - x^2)/(x^2 + 1)^2$**.
4. $dy/dx = e^{-2x} - 2xe^{-2x} = e^{-2x}(1 - 2x) = 0$ at $x = 1/2$. $y = (1/2)e^{-1}$ = **0.184**, so the point is **(0.5, 0.184)**.
5. $2x + 3y + 3x(dy/dx) + 2y(dy/dx) = 0$, so $dy/dx = -(2x + 3y)/(3x + 2y) = -8/7$. **$dy/dx = -8/7$**.
6. $dV/dr = 4\pi r^2 = 100\pi$. $dV/dt = 100\pi \times 0.2 = 20\pi$ = **62.8 cm³/s**.