Functions, the Modulus and Partial Fractions

Pure: Algebra and functions · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$fg(x) = f(g(x))$ $|x - a| < b \Leftrightarrow a - b < x < a + b$ $N(x)/[(x - a)(x - b)] = A/(x - a) + B/(x - b)$

domain of f^{-1} = range of f

Questions

Show your working in the space under each question.

1. EASY Find the inverse of $f(x) = (x - 4)/3$.

2. EASY State the range of $f(x) = x^2 + 2$ for all real x.

3. MEDIUM Solve $|x + 2| = 3x$.

4. MEDIUM $f(x) = \sqrt{(x - 1)}$ for $x \geq 1$. Find $f^{-1}(x)$ and its domain.

5. HARD Express $3/[x(x + 3)]$ in partial fractions.

6. HARD $f(x) = x - 1$ and $g(x) = x^2 + 3x$. Solve $gf(x) = 4$.

Step-by-step answers

Functions, the Modulus and Partial Fractions

1. $y = (x - 4)/3$ gives $x = 3y + 4$. $\mathbf{f^{-1}(x) = 3x + 4}$.
2. $x^2 \geq 0$, so $\mathbf{f(x) \geq 2}$.
3. Case 1: $x + 2 = 3x$ gives $x = 1$. Case 2: $-(x + 2) = 3x$ gives $x = -1/2$, but then $3x$ is negative and cannot equal a modulus. $\mathbf{x = 1}$ only.
4. $y = \sqrt{(x - 1)}$ gives $x = y^2 + 1$. $\mathbf{f^{-1}(x) = x^2 + 1}$**, with domain** $\mathbf{x \geq 0}$ (the range of f).
5. $3 = A(x + 3) + Bx$. $x = 0$: $A = 1$. $x = -3$: $B = -1$. $\mathbf{1/x - 1/(x + 3)}$.
6. $gf(x) = (x - 1)^2 + 3(x - 1) = x^2 + x - 2$. Set equal to 4: $x^2 + x - 6 = 0$, so $\mathbf{x = 2}$ **or** $\mathbf{x = -3}$.