Numerical Methods

Pure: Numerical methods · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$x_{n+1} = g(x_n)$ $x_{n+1} = x_n - f(x_n)/f'(x_n)$ $\int \approx (h/2)[y_0 + 2(y_1 + \ldots + y_{n-1}) + y_n]$

$f(a)f(b) < 0 \Rightarrow$ root in (a, b)

Questions

Show your working in the space under each question.

1. EASY Show that $x^2 - 5 = 0$ has a root between 2 and 3.

2. EASY Use the trapezium rule with 2 strips to estimate $\int_0^2 x^3\,dx$.

3. MEDIUM Use $x_{n+1} = (x_n^2 + 2)/5$ with $x_0 = 0$ to find a root of $x^2 - 5x + 2 = 0$ to 2 decimal places.

4. MEDIUM Use Newton–Raphson on $f(x) = x^2 - 2$ from $x_0 = 1$ to find $\sqrt{2}$ to 4 decimal places.

5. HARD Use the trapezium rule with 4 strips to estimate $\int_1^3 (1/x)\,dx$.

6. HARD Explain why Newton–Raphson fails for $f(x) = x^3 - 3x + 1$ with $x_0 = 1$.

Step-by-step answers

Numerical Methods

1. $f(2) = -1$ and $f(3) = 4$: a sign change in a continuous function, so **there is a root in (2, 3)**.
2. $h = 1$, ordinates 0, 1, 8. $(1/2)[0 + 2(1) + 8] =$ **5** (the exact value is 4; the curve is concave up, so this is an overestimate).
3. $x_1 = 0.4$, $x_2 = 0.432$, $x_3 = 0.4373$, $x_4 = 0.4382$. **$x \approx 0.44$**.
4. $x_{n+1} = x_n - (x_n^2 - 2)/(2x_n)$: 1.5, 1.4167, 1.4142. **$\sqrt{2} \approx 1.4142$**.
5. $h = 0.5$, ordinates 1, 0.6667, 0.5, 0.4, 0.3333. $(0.5/2)[1 + 0.3333 + 2(1.5667)] = 0.25 \times 4.4667 =$ **1.117** (exact: $\ln 3 = 1.099$).
6. $f'(x) = 3x^2 - 3$, so $f'(1) = 0$. The formula divides by zero: **the tangent at x = 1 is horizontal and never meets the x-axis**.