Sequences, Series and the General Binomial Expansion

Pure: Sequences and series · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$u_n = a + (n-1)d$, $S_n = (n/2)[2a + (n-1)d]$ $u_n = ar^{n-1}$, $S_n = a(1-r^n)/(1-r)$

$S_\infty = a/(1-r)$, $|r| < 1$ $(1+x)^n = 1 + nx + n(n-1)x^2/2! + \ldots$, $|x| < 1$

Questions

Show your working in the space under each question.

1. EASY Find the sum of the first 20 terms of 2, 5, 8, ...

2. EASY Find the 7th term of 5, 15, 45, ...

3. MEDIUM The 5th term of an arithmetic sequence is 17 and the 9th is 33. Find a and d.

4. MEDIUM Find the sum to infinity of $12 + 4 + 4/3 + \ldots$

5. HARD Expand $\sqrt{(1 + x)}$ up to x^2 and use it to estimate $\sqrt{1.04}$.

6. HARD A geometric series has first term 8 and sum to infinity 20. Find r and the least number of terms whose sum exceeds 19.

Step-by-step answers

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1. $a = 2$, $d = 3$. $S_{20} = 10[4 + 19 \times 3] = 10 \times 61 = \mathbf{610}$.

2. $r = 3$: $5 \times 3^6 = 5 \times 729 = \mathbf{3645}$.

3. $4d = 16$, so $\mathbf{d = 4}$. Then $a + 16 = 17$, so $\mathbf{a = 1}$.

4. $r = 1/3$. $S_\infty = 12/(2/3) = \mathbf{18}$.

5. $1 + x/2 + [(1/2)(-1/2)/2]x^2 = 1 + x/2 - x^2/8$. With $x = 0.04$: $1 + 0.02 - 0.0002 = \mathbf{1.0198}$.

6. $8/(1 - r) = 20$ gives $\mathbf{r = 0.6}$. Need $20(1 - 0.6^n) > 19$, so $0.6^n < 0.05$ and $n > \ln 0.05/\ln 0.6 = 5.86$. $\mathbf{n = 6}$.