Statistics: the Normal Distribution and Hypothesis Tests

Statistics · 6 questions · step-by-step answers at the end

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USEFUL EQUATIONS

$z = (x - \mu)/\sigma$ $P(X < x) = \Phi(z)$ $\bar{X} \sim N(\mu, \sigma^2/n)$ $B(n, p) \approx N(np, np(1 - p))$

Questions

Show your working in the space under each question.

1. EASY $X \sim N(70, 8^2)$. Find the z-value for $x = 82$.

2. EASY Find $P(Z < -1)$.

3. MEDIUM Heights are $N(170, 6^2)$ in cm. Find the probability that a person is taller than 179 cm.

4. MEDIUM Scores are $N(100, 15^2)$. What score is exceeded by only the top 5%?

5. HARD $X \sim B(100, 0.5)$. Use a normal approximation to find $P(X \geq 60)$.

6. HARD A machine is set to a mean of 50 with $\sigma = 4$. A sample of 25 has mean 51.8. Test at the 5% level whether the mean has increased.

Step-by-step answers

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1. $z = (82 - 70)/8 = \mathbf{1.5}$.
2. By symmetry, $1 - \Phi(1) = 1 - 0.8413 = \mathbf{0.1587}$.
3. $z = 1.5$. $P(Z > 1.5) = 1 - 0.9332 = \mathbf{0.0668}$.
4. $\Phi(z) = 0.95$ gives $z = 1.645$. $x = 100 + 1.645 \times 15 = \mathbf{124.7}$.
5. Approximate by $N(50, 25)$. With continuity correction: $z = (59.5 - 50)/5 = 1.9$. $P(Z > 1.9) = \mathbf{0.0287}$.
6. H_0: $\mu = 50$, H_1: $\mu > 50$. Standard error $= 4/\sqrt{25} = 0.8$. $z = 1.8/0.8 = 2.25 > 1.645$, so **reject H_0: there is evidence the mean has increased**.