Radians and Further Trigonometry

Pure: Trigonometry · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$s = r\theta$, $A = (1/2)r^2\theta$ $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$ $a \sin x + b \cos x = R \sin(x + \alpha)$

Questions

Show your working in the space under each question.

1. EASY Convert $2\pi/3$ radians to degrees.

2. EASY A sector has radius 5 cm and angle 0.8 radians. Find its perimeter.

3. MEDIUM Solve $\sin 2x = \sin x$ for $0° \le x \le 360°$.

4. MEDIUM Solve $\sec x = 2$ for $0 \le x \le 2\pi$.

5. HARD Solve $\cos 2x + \sin x = 0$ for $0° \leq x \leq 360°$.

6. HARD Use small-angle approximations to simplify $(1 - \cos 2\theta)/(\theta \sin 3\theta)$.

Step-by-step answers

Radians and Further Trigonometry

1. $2/3 \times 180 = \mathbf{120°}$.
2. Arc $= 5 \times 0.8 = 4$. Perimeter $= 5 + 5 + 4 = \mathbf{14\ cm}$.
3. $2 \sin x \cos x - \sin x = 0$, so $\sin x(2 \cos x - 1) = 0$. $\sin x = 0$: 0°, 180°, 360°. $\cos x = 1/2$: 60°, 300°. $\mathbf{x = 0°, 60°, 180°, 300°, 360°}$.
4. $\cos x = 1/2$, so $\mathbf{x = \pi/3 \text{ or } 5\pi/3}$.
5. Use $\cos 2x = 1 - 2 \sin^2 x$: $2 \sin^2 x - \sin x - 1 = 0$, so $(2 \sin x + 1)(\sin x - 1) = 0$. $\mathbf{x = 90°, 210°, 330°}$.
6. $1 - \cos 2\theta \approx (2\theta)^2/2 = 2\theta^2$, and $\sin 3\theta \approx 3\theta$. So the expression $\approx 2\theta^2/(3\theta^2) = \mathbf{2/3}$.