Vectors in Three Dimensions

Pure: Vectors · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$|a| = \sqrt{(a_1^2 + a_2^2 + a_3^2)}$ $a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$ $\cos\theta = (a \cdot b)/(|a||b|)$ $AB = b - a$

Questions

Show your working in the space under each question.

1. EASY Work out $(1, 2, 3) + 2(0, -1, 4)$.

2. EASY Find the distance between A(1, 0, 2) and B(3, 3, 8).

3. MEDIUM Show that $(2, 3, -1)$ and $(4, -2, 2)$ are perpendicular.

4. MEDIUM $(p, 2, 1)$ is perpendicular to $(3, -4, 2)$. Find p.

5. HARD A(0, 3, 6) and B(9, 0, 0). Find the point P on AB with AP : PB = 1 : 2.

6. HARD Find the angle between (1, 1, 0) and (0, 1, 1).

Step-by-step answers

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1. $(1, 2 - 2, 3 + 8) = \mathbf{(1, 0, 11)}$.

2. $\text{AB} = (2, 3, 6)$, $|\text{AB}| = \sqrt{(4 + 9 + 36)} = \mathbf{7}$.

3. Dot product: $8 - 6 - 2 = 0$, so **they are perpendicular**.

4. $3p - 8 + 2 = 0$, so $\mathbf{p = 2}$.

5. $\text{OP} = \text{a} + (1/3)(\text{b} - \text{a}) = (0, 3, 6) + (3, -1, -2) = \mathbf{(3, 2, 4)}$.

6. Dot product 1; each magnitude is $\sqrt{2}$. $\cos\theta = 1/2$, so $\boldsymbol{\theta = 60°}$.