Quadratics, Inequalities and the Discriminant

Pure: Algebra and functions · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\Delta = b^2 - 4ac$ $x^2 + bx + c = (x + b/2)^2 - b^2/4 + c$ $\Delta = 0 \Leftrightarrow$ tangent (repeated root)

$(x - \alpha)(x - \beta) < 0 \Leftrightarrow \alpha < x < \beta$

Questions

Show your working in the space under each question.

1. EASY Write $x^2 + 8x + 3$ in completed-square form.

2. EASY Simplify $\sqrt{75} - \sqrt{27}$.

3. MEDIUM Solve $2x^2 - 5x - 3 \leq 0$.

4. MEDIUM Find the values of k for which $x^2 + 6x + k = 0$ has two distinct real roots.

5. HARD Solve $x^4 - 5x^2 + 4 = 0$.

6. HARD Find the values of k for which $kx^2 + 4x + k = 0$ has no real roots.

Step-by-step answers

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1. $(x + 4)^2 - 16 + 3 = \mathbf{(x + 4)^2 - 13}$.
2. $5\sqrt{3} - 3\sqrt{3} = \mathbf{2\sqrt{3}}$.
3. $(2x + 1)(x - 3) \leq 0$. Critical values $-1/2$ and 3; below the axis between them: $\mathbf{-1/2 \leq x \leq 3}$.
4. $\Delta = 36 - 4k > 0$, so $\mathbf{k < 9}$.
5. Let $u = x^2$: $u^2 - 5u + 4 = 0$, so $u = 1$ or 4. $\mathbf{x = \pm 1 \text{ or } \pm 2}$.
6. $\Delta = 16 - 4k^2 < 0$, so $k^2 > 4$. $\mathbf{k > 2 \text{ or } k < -2}$.