Differentiation

Pure: Calculus · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$y = x^n \Rightarrow dy/dx = nx^{n-1}$ $y - y_1 = m(x - x_1)$ $dy/dx = 0$ $d^2y/dx^2 > 0 \Rightarrow$ minimum

Questions

Show your working in the space under each question.

1. EASY Differentiate $y = 5x^4 + 3x$.

2. EASY Differentiate $y = 4\sqrt{x} + 2/x$.

3. MEDIUM Find the gradient of $y = x^2 + 1/x$ at $x = 1$.

4. MEDIUM For which values of x is $y = x^2 - 8x$ increasing?

5. HARD Find the normal to $y = x^3$ at the point (1, 1).

6. HARD A rectangular pen is built against a wall using 40 m of fence for the other three sides. Find the largest possible area.

Step-by-step answers

Differentiation

1. **$dy/dx = 20x^3 + 3$**.
2. $y = 4x^{1/2} + 2x^{-1}$, so $dy/dx = 2x^{-1/2} - 2x^{-2}$ = **$2/\sqrt{x} - 2/x^2$**.
3. $dy/dx = 2x - 1/x^2 = 2 - 1$ = **1**.
4. $dy/dx = 2x - 8 > 0$ when **$x > 4$**.
5. Tangent gradient $3x^2 = 3$, so normal gradient $-1/3$: **$y - 1 = -(1/3)(x - 1)$**.
6. Sides x, x and $40 - 2x$. $A = 40x - 2x^2$, $dA/dx = 40 - 4x = 0$ at $x = 10$. **$A = 10 \times 20 = 200$ m²**.