Exponentials and Logarithms

Pure: Exponentials and logarithms · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\log_a(xy) = \log_a x + \log_a y \quad \log_a(x/y) = \log_a x - \log_a y \quad \log_a(x^k) = k \log_a x$

$a^x = b \Rightarrow x = \ln b / \ln a$

Questions

Show your working in the space under each question.

1. EASY Evaluate $\log_3 81$.

2. EASY Simplify $\log_{10} 2 + \log_{10} 5$.

3. MEDIUM Solve $e^{2x} = 7$.

4. MEDIUM Solve $2 \log x - \log 4 = \log 9$ for $x > 0$.

5. HARD A mass decays as $m = 80e^{-0.05t}$ (t in days). Find its half-life.

6. HARD Solve $2^{2x} - 5(2^x) + 4 = 0$.

Step-by-step answers

Exponentials and Logarithms

1. $3^4 = 81$, so **4**.

2. $\log_{10}(2 \times 5) = \log_{10} 10 = \mathbf{1}$.

3. $2x = \ln 7$, so $x = (\ln 7)/2 = \mathbf{0.973}$.

4. $\log(x^2/4) = \log 9$, so $x^2 = 36$ and $\mathbf{x = 6}$.

5. $0.5 = e^{-0.05t}$, so $t = \ln 2/0.05 = \mathbf{13.9\ days}$.

6. Let $u = 2^x$: $u^2 - 5u + 4 = 0$, so $u = 1$ or 4. $\mathbf{x = 0 \text{ or } x = 2}$.