Integration

Pure: Calculus · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\int x^n\,dx = x^{n+1}/(n+1) + c$ $\int_a^b f(x)\,dx = F(b) - F(a)$ $\text{Area} = \int_a^b (\text{top} - \text{bottom})\,dx$

$y = \int (dy/dx)\,dx$

Questions

Show your working in the space under each question.

1. EASY Find $\int 6x^5\,dx$.

2. EASY Find $\int (x^2 - 2/x^2)\,dx$.

3. MEDIUM Evaluate $\int_0^3 (x^2 + 1)\,dx$.

4. MEDIUM A curve has $dy/dx = 4x - 3$ and passes through $(1, 2)$. Find its equation.

5. HARD Find the area under $y = x(4 - x)$ between $x = 0$ and $x = 4$.

6. HARD Find the area enclosed by $y = x^2 - 4$ and the x-axis.

Step-by-step answers

Integration

1. $\mathbf{x^6 + c}$.
2. $x^3/3 - 2(x^{-1}/(-1)) = \mathbf{x^3/3 + 2/x + c}$.
3. $[x^3/3 + x]$ from 0 to 3 $= 9 + 3 = \mathbf{12}$.
4. $y = 2x^2 - 3x + c$. At (1, 2): $2 - 3 + c = 2$, so $c = 3$. $\mathbf{y = 2x^2 - 3x + 3}$.
5. $\int_0^4 (4x - x^2)\,dx = [2x^2 - x^3/3] = 32 - 64/3 = \mathbf{32/3}$.
6. Roots ± 2. $\int_{-2}^{2} (x^2 - 4)\,dx = -32/3$. The region is below the axis, so the area is $\mathbf{32/3}$.