Polynomials, the Factor Theorem and Binomial Expansion

Pure: Algebra and functions · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$f(a) = 0 \Leftrightarrow (x - a)$ is a factor $\quad (a + b)^n = \Sigma\, C(n, r)\, a^{n-r}\, b^r \quad C(n, r) = n!/[r!(n - r)!]$

$(1 + x)^n = 1 + nx + n(n - 1)x^2/2 + \ldots$

Questions

Show your working in the space under each question.

1. EASY Find the remainder when $x^3 + 2x - 5$ is divided by $(x - 1)$.

2. EASY Expand $(1 + x)^4$.

3. MEDIUM $(x + 1)$ is a factor of $x^3 + ax^2 - x + 2$. Find a.

4. MEDIUM Find the first three terms of $(1 - 2x)^6$.

5. HARD Solve $x^3 - 6x^2 + 11x - 6 = 0$.

6. HARD Find the coefficient of x^3 in $(3 - x)^5$.

Step-by-step answers

Polynomials, the Factor Theorem and Binomial Expansion

1. $f(1) = 1 + 2 - 5 = \mathbf{-2}$.
2. Coefficients 1, 4, 6, 4, 1: $\mathbf{1 + 4x + 6x^2 + 4x^3 + x^4}$.
3. $f(-1) = -1 + a + 1 + 2 = 0$, so $\mathbf{a = -2}$.
4. $1 + 6(-2x) + 15(-2x)^2 = \mathbf{1 - 12x + 60x^2}$.
5. $f(1) = 0$, so $(x - 1)$ is a factor: $(x - 1)(x^2 - 5x + 6) = (x - 1)(x - 2)(x - 3)$. $\mathbf{x = 1, 2 \text{ or } 3}$.
6. $C(5, 3) \times 3^2 \times (-1)^3 = 10 \times 9 \times (-1) = \mathbf{-90}$.