Statistics: Data, Probability and the Binomial Distribution

Statistics · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$P(X = r) = C(n, r)\, p^r\, (1 - p)^{n-r}$ $E(X) = np$, $Var(X) = np(1 - p)$ $\sigma^2 = \Sigma(x - \bar{x})^2/n$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Questions

Show your working in the space under each question.

1. EASY Find the variance and standard deviation of 2, 4, 6, 8.

2. EASY A and B are independent with $P(A) = 0.4$ and $P(B) = 0.5$. Find $P(A \cap B)$ and $P(A \cup B)$.

3. MEDIUM $X \sim B(6, 1/3)$. Find $P(X = 2)$.

4. MEDIUM $X \sim B(8, 0.25)$. Find the mean and variance.

5. HARD $X \sim B(4, 0.3)$. Find $P(X \leq 1)$.

6. HARD A random variable has $P(X = x) = kx$ for $x = 1, 2, 3, 4$. Find k and $E(X)$.

Step-by-step answers

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1. Mean 5. Squared deviations 9, 1, 1, 9, mean 5. **Variance 5, standard deviation 2.24**.

2. $P(A \cap B) = 0.4 \times 0.5 = \mathbf{0.2}$. $P(A \cup B) = 0.4 + 0.5 - 0.2 = \mathbf{0.7}$.

3. $C(6, 2)(1/3)^2(2/3)^4 = 15 \times (1/9) \times (16/81) = 240/729 = \mathbf{0.329}$.

4. Mean $= 8 \times 0.25 = \mathbf{2}$. Variance $= 8 \times 0.25 \times 0.75 = \mathbf{1.5}$.

5. $P(0) = 0.7^4 = 0.2401$. $P(1) = 4 \times 0.3 \times 0.7^3 = 0.4116$. Total **0.6517**.

6. $k(1 + 2 + 3 + 4) = 1$, so $\mathbf{k = 1/10}$. $E(X) = (1 + 4 + 9 + 16)/10 = \mathbf{3}$.