Antiderivatives and u-Substitution

Integration and Accumulation of Change · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\int x^n\,dx = x^{n+1}/(n+1) + C \quad \int (1/x)\,dx = \ln|x| + C$

$\int e^x\,dx = e^x + C,\ \int \cos x\,dx = \sin x + C,\ \int \sin x\,dx = -\cos x + C \quad \int f(g(x))\,g'(x)\,dx = \int f(u)\,du$

Questions

Show your working in the space under each question.

1. EASY Find $\int (1/x^2 + \sqrt{x})\,dx$.

2. EASY Find $\int (\cos x - 3e^x)\,dx$.

3. MEDIUM Find $\int xe^{x^2}\,dx$.

4. MEDIUM $f'(x) = 3x^2 - 2$ and $f(1) = 4$. Find $f(x)$.

5. HARD Find $\int (\ln x)^2/x \, dx$.

6. HARD Find $\int x/\sqrt{(2x + 1)} \, dx$.

Step-by-step answers

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1. Rewrite as $x^{-2} + x^{1/2}$. Integrate: $\mathbf{-1/x + (2/3)x^{3/2} + C}$.
2. $\mathbf{\sin x - 3e^{x} + C}$.
3. Let $u = x^2$, $du = 2x\,dx$, so $x\,dx = du/2$. $\int (1/2)e^{u}\,du = \mathbf{(1/2)e^{x^2} + C}$.
4. $f(x) = x^3 - 2x + C$. Use $f(1) = 4$: $1 - 2 + C = 4$, so $C = 5$. $\mathbf{f(x) = x^3 - 2x + 5}$.
5. Let $u = \ln x$, $du = dx/x$. $\int u^2\,du = u^3/3 + C = \mathbf{(\ln x)^3/3 + C}$.
6. Let $u = 2x + 1$, so $dx = du/2$ and $x = (u - 1)/2$. The integral is $(1/4)\int (u^{1/2} - u^{-1/2})\,du = (1/4)[(2/3)u^{3/2} - 2u^{1/2}] + C = \mathbf{(1/6)(2x + 1)^{3/2} - (1/2)(2x + 1)^{1/2} + C}$.