Arc Length

Applications of Integration (BC) · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$L = \int_a^b \sqrt{(1 + (dy/dx)^2)}\, dx \quad L = \int_c^d \sqrt{(1 + (dx/dy)^2)}\, dy \quad ds = \sqrt{(dx^2 + dy^2)}$

$L = \int \sqrt{((dx/dt)^2 + (dy/dt)^2)}\, dt$

Questions

Show your working in the space under each question.

1. EASY Find the length of $y = 3x - 2$ from $x = 1$ to $x = 5$.

2. EASY Set up (do not evaluate) the integral for the length of $y = x^2$ from $x = 0$ to $x = 1$.

3. MEDIUM Find the length of $y = x^{3/2}$ from $x = 0$ to $x = 4$.

4. MEDIUM Find the length of $y = \ln(\cos x)$ from $x = 0$ to $x = \pi/4$.

5. HARD Find the length of $y = (e^x + e^{-x})/2$ from $x = 0$ to $x = 1$.

6. HARD Use the arc length formula to show that a circle of radius r has circumference $2\pi r$.

Step-by-step answers

Arc Length

1. $\sqrt{(1 + 9)} = \sqrt{10}$. $L = 4\sqrt{10} \approx \mathbf{12.65}$.
2. $dy/dx = 2x$, so $\mathbf{L = \int_0^1 \sqrt{(1 + 4x^2)}\, dx}$ (a calculator gives 1.479).
3. $dy/dx = (3/2)x^{1/2}$, so $L = \int_0^4 \sqrt{(1 + 9x/4)}\, dx = (8/27)[(1 + 9x/4)^{3/2}] = (8/27)(10^{3/2} - 1) \approx \mathbf{9.07}$.
4. $dy/dx = -\tan x$, and $1 + \tan^2 x = \sec^2 x$. $L = \int_0^{\pi/4} \sec x\, dx = [\ln(\sec x + \tan x)] = \ln(\sqrt{2} + 1) \approx \mathbf{0.881}$.
5. $dy/dx = (e^x - e^{-x})/2$, and $1 + (dy/dx)^2 = [(e^x + e^{-x})/2]^2$. $L = \int_0^1 (e^x + e^{-x})/2\, dx = (e - 1/e)/2 \approx \mathbf{1.175}$.
6. Top-right quarter: $y = \sqrt{(r^2 - x^2)}$, $dy/dx = -x/\sqrt{(r^2 - x^2)}$, so $1 + (dy/dx)^2 = r^2/(r^2 - x^2)$. $L = \int_0^r r/\sqrt{(r^2 - x^2)}\, dx = r[\sin^{-1}(x/r)] = r\pi/2$. Four quarters: $\mathbf{2\pi r}$.