Area Between Curves

Applications of Integration · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$A = \int_a^b [f(x) - g(x)]\,dx$ $A = \int_c^d [\text{right} - \text{left}]\,dy$ $f(x) = g(x)$ $A = \int_a^b |f(x) - g(x)|\,dx$

Questions

Show your working in the space under each question.

1. EASY Find the area between $y = 4 - x^2$ and the x-axis.

2. EASY Find the area between $y = 2x$ and $y = x^2$.

3. MEDIUM Find the total area between $y = x^3$ and $y = x$.

4. MEDIUM Find the area between $y = e^x$ and $y = x$ from $x = 0$ to $x = 1$.

5. HARD Find the area between $x = y^2$ and $x = y + 2$ by integrating with respect to y.

6. HARD Find the area between $y = x^2 - 4$ and $y = 4 - x^2$.

Step-by-step answers

Area Between Curves

1. Roots at $x = \pm 2$. $\int_{-2}^{2} (4 - x^2)\,dx = [4x - x^3/3] = (8 - 8/3) - (-8 + 8/3) = \mathbf{32/3}$.
2. Intersections at $x = 0$ and 2, line on top. $\int_{0}^{2} (2x - x^2)\,dx = [x^2 - x^3/3] = 4 - 8/3 = \mathbf{4/3}$.
3. They meet at -1, 0 and 1. On (0, 1) the line is on top: $\int_{0}^{1} (x - x^3)\,dx = 1/2 - 1/4 = 1/4$. By symmetry the other region is also 1/4. Total **1/2**.
4. e^x is on top. $\int_{0}^{1} (e^x - x)\,dx = [e^x - x^2/2] = (e - 1/2) - 1 = \mathbf{e - 1.5 \approx 1.218}$.
5. $y^2 = y + 2$ gives $y = -1$ and 2. The line is on the right. $\int_{-1}^{2} (y + 2 - y^2)\,dy = \mathbf{9/2}$.
6. They meet at $x = \pm 2$, with $4 - x^2$ on top. $\int_{-2}^{2} (8 - 2x^2)\,dx = [8x - 2x^3/3] = 2(16 - 16/3) = \mathbf{64/3}$.