Continuity, Asymptotes and the Intermediate Value Theorem

Limits and Continuity · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\lim_{x \to a} f(x) = f(a)$ $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ $y = \lim_{x \to \pm\infty} f(x)$

$f(a) < N < f(b) \Rightarrow f(c) = N$ for some c in (a, b)

Questions

Show your working in the space under each question.

1. EASY Is $f(x) = 1/(x - 5)$ continuous at $x = 5$? Explain.

2. EASY $g(x) = 2x + 1$ for $x < 4$ and $g(x) = 9$ for $x \geq 4$. Is g continuous at $x = 4$?

3. MEDIUM $f(x) = cx^2 + 1$ for $x \leq 1$ and $f(x) = 4x - c$ for $x > 1$. Find c so that f is continuous.

4. MEDIUM Find all asymptotes of $y = (3x^2 + 1)/(x^2 - 4)$.

5. HARD Show that $\cos x = x$ has a solution between 0 and $\pi/2$.

6. HARD $f(x) = x + 1$ for $x < 1$, $f(x) = ax + b$ for $1 \leq x \leq 3$, and $f(x) = x^2$ for $x > 3$. Find a and b so that f is continuous.

Step-by-step answers

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1. No. f(5) is undefined and the function grows without bound near 5, so there is an infinite discontinuity (vertical asymptote) at x = 5.

2. Left-hand limit: 2(4) + 1 = 9. Right-hand limit: 9. g(4) = 9. All three agree, so **yes**, g is continuous at 4.

3. At x = 1: left value c + 1, right limit 4 − c. Set equal: c + 1 = 4 − c, so 2c = 3 and c = **3/2**.

4. Bottom = (x − 2)(x + 2), and neither factor cancels, so vertical asymptotes at **x = 2 and x = −2**. Equal degrees, so the horizontal asymptote is the ratio of leading coefficients: **y = 3**.

5. Let g(x) = cos x − x, which is continuous everywhere. g(0) = 1 > 0 and g(π/2) = 0 − π/2 < 0. g changes sign, so by the IVT there is a c in (0, π/2) with g(c) = 0, that is cos c = c.

6. At x = 1: 1 + 1 = a + b, so a + b = 2. At x = 3: 3a + b = 9. Subtract: 2a = 7, so **a = 7/2**, and b = 2 − 7/2 = **−3/2**.