Extrema, Concavity and Curve Sketching

Analytical Applications of Differentiation · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$f'(x) > 0 \Rightarrow$ increasing; $f'(x) < 0 \Rightarrow$ decreasing

$f''(x) > 0 \Rightarrow$ concave up; $f''(x) < 0 \Rightarrow$ concave down $f'(c) = 0$ and $f''(c) > 0 \Rightarrow$ local minimum

$f'(c) = 0$ and $f''(c) < 0 \Rightarrow$ local maximum

Questions

Show your working in the space under each question.

1. EASY On what interval is $f(x) = x^2 + 4x$ increasing?

2. EASY Find and classify the critical points of $f(x) = x^3 - 12x$.

3. MEDIUM Find where $f(x) = x^4 - 6x^2$ is concave down, and its inflection points.

4. MEDIUM Find the absolute maximum and minimum of $f(x) = x^3 - 3x + 1$ on $[0, 2]$.

5. HARD Use the second derivative test on $f(x) = x + 4/x$ for $x > 0$.

6. HARD The derivative of f is $f'(x) = (x - 1)^2(x + 2)$. At which x does f have a local extremum, and of what type?

Step-by-step answers

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1. $f' = 2x + 4 > 0$ when **$x > -2$**.

2. $f' = 3x^2 - 12 = 3(x - 2)(x + 2)$. Sign: +, −, +. **Local max at (−2, 16)**, **local min at (2, −16)**.

3. $f'' = 12x^2 - 12 = 12(x - 1)(x + 1)$, negative for **$-1 < x < 1$**. The concavity changes at $x = \pm 1$, where $f = 1 - 6 = -5$: inflection points **(−1, −5) and (1, −5)**.

4. $f' = 3x^2 - 3 = 0$ at $x = 1$ ($x = -1$ is outside). $f(0) = 1$, $f(1) = -1$, $f(2) = 3$. **Max 3 at $x = 2$, min −1 at $x = 1$**.

5. $f' = 1 - 4/x^2 = 0$ at $x = 2$. $f'' = 8/x^3$, and $f''(2) = 1 > 0$, so there is a **local minimum at $x = 2$ with $f(2) = 4$**.

6. $f' = 0$ at $x = 1$ and $x = -2$. $(x - 1)^2$ is never negative, so the sign of f' is the sign of $(x + 2)$: − before −2 and + after. **Local minimum at $x = -2$**. At $x = 1$ the sign does not change, so there is no extremum there.