The Derivative as a Limit

Differentiation: Definition and Basic Rules · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$f'(x) = \lim_{h\to 0} [f(x+h) - f(x)]/h \quad f'(a) = \lim_{x\to a} [f(x) - f(a)]/(x - a) \quad [f(b) - f(a)]/(b - a)$

$y - f(a) = f'(a)(x - a)$

Questions

Show your working in the space under each question.

1. EASY Find the average rate of change of $f(x) = x^2 + 1$ on $[1, 3]$.

2. EASY Use the limit definition to differentiate $f(x) = 3x - 7$.

3. MEDIUM Use the limit definition to differentiate $f(x) = x^2 + 2x$.

4. MEDIUM Find the tangent line to $y = x^2$ at $x = -1$.

5. HARD Evaluate $\lim_{h \to 0} [(1 + h)^{10} - 1]/h$ without expanding.

6. HARD Use the limit definition to differentiate $f(x) = x^3$.

Step-by-step answers

The Derivative as a Limit

1. $[f(3) - f(1)]/(3 - 1) = (10 - 2)/2 = \mathbf{4}$.

2. $[3(x + h) - 7 - (3x - 7)]/h = 3h/h = 3$. The limit is **3**, the slope of the line.

3. $[(x + h)^2 + 2(x + h) - x^2 - 2x]/h = (2xh + h^2 + 2h)/h = 2x + h + 2$. Let $h \to 0$: $\mathbf{f'(x) = 2x + 2}$.

4. Slope: $f'(x) = 2x$, so $f'(-1) = -2$. Point: $(-1, 1)$. Line: $y - 1 = -2(x + 1)$, that is $\mathbf{y = -2x - 1}$.

5. This is the definition of $f'(1)$ for $f(x) = x^{10}$. Since $f'(x) = 10x^9$, the limit is $10(1)^9 = \mathbf{10}$.

6. $(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$. Subtract x^3 and divide by h: $3x^2 + 3xh + h^2$. Let $h \to 0$: $\mathbf{f'(x) = 3x^2}$.