Power, Product, Quotient and Chain Rules

Differentiation: Definition and Basic Rules · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\frac{d}{dx}(x^n) = nx^{n-1}$ $(uv)' = u'v + uv'$ $(u/v)' = (u'v - uv')/v^2$ $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Questions

Show your working in the space under each question.

1. EASY Differentiate $y = 3x^4 - 5x + 9$.

2. EASY Differentiate $y = 6/x^2 + \sqrt{x}$.

3. MEDIUM Differentiate $y = (3x + 1)^4$.

4. MEDIUM Differentiate $y = x/(x + 2)$.

5. HARD Differentiate $y = x^2\sqrt{(1 + x^2)}$ and simplify.

6. HARD Given $f(2) = 1$, $f'(2) = 2$, $f'(3) = 4$, $g(2) = 3$ and $g'(2) = -5$, find $h'(2)$ for $h(x) = f(g(x))$ and $k'(2)$ for $k(x) = f(x)g(x)$.

Step-by-step answers

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1. Term by term: $\mathbf{y' = 12x^3 - 5}$.
2. Rewrite: $y = 6x^{-2} + x^{1/2}$. Then $y' = -12x^{-3} + (1/2)x^{-1/2} = \mathbf{-12/x^3 + 1/(2\sqrt{x})}$.
3. Chain rule: outside power 4, inside $3x + 1$ with derivative 3. $y' = 4(3x + 1)^3 \cdot 3 = \mathbf{12(3x + 1)^3}$.
4. Quotient rule with $u = x$, $v = x + 2$: $y' = [1(x + 2) - x(1)]/(x + 2)^2 = \mathbf{2/(x + 2)^2}$.
5. Product + chain: $y' = 2x\sqrt{(1 + x^2)} + x^2 \cdot x/\sqrt{(1 + x^2)}$. Put over a common denominator: $[2x(1 + x^2) + x^3]/\sqrt{(1 + x^2)} = \mathbf{(2x + 3x^3)/\sqrt{(1 + x^2)}}$.
6. Chain rule: $h'(2) = f'(g(2)) \cdot g'(2) = f'(3) \cdot (-5) = 4(-5) = \mathbf{-20}$. Product rule: $k'(2) = f'(2)g(2) + f(2)g'(2) = 2(3) + 1(-5) = \mathbf{1}$.