Differential Equations and Slope Fields

Differential Equations · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$dy/dx = f(x)g(y) \Rightarrow \int dy/g(y) = \int f(x)\,dx$ $dy/dt = ky \Rightarrow y = y_0 e^{kt}$ $t_{1/2} = (\ln 2)/|k|$

$T - T_s = (T_0 - T_s)e^{-kt}$

Questions

Show your working in the space under each question.

1. EASY Solve $dy/dx = \cos x$ with $y(0) = 1$.

2. EASY For $dy/dx = x - y$, find the slope at $(2, 1)$ and describe where the slopes are zero.

3. MEDIUM Solve $dy/dx = y^2$ with $y(0) = 1$.

4. MEDIUM A population obeys $dP/dt = kP$. It grows from 500 to 1000 in 4 hours. Find the population after 10 hours.

5. HARD Solve $dy/dx = x/y$ with $y(0) = -2$.

6. HARD Solve $dy/dx = 2x(1 + y^2)$ with $y(0) = 0$.

Step-by-step answers

Differential Equations and Slope Fields

1. $y = \sin x + C$, and $y(0) = 1$ gives $C = 1$. **$y = \sin x + 1$**.

2. Slope $= 2 - 1 =$ **1**. Slopes are zero where $x - y = 0$, along the line **$y = x$**.

3. Separate: $y^{-2}\,dy = dx$. Integrate: $-1/y = x + C$. $y(0) = 1$ gives $C = -1$. So $-1/y = x - 1$ and **$y = 1/(1 - x)$**.

4. It doubles every 4 hours, so $P = 500 \cdot 2^{t/4}$. $P(10) = 500 \cdot 2^{2.5} \approx$ **2828**.

5. Separate: $y\,dy = x\,dx$. Integrate: $y^2/2 = x^2/2 + C$, so $y^2 = x^2 + 4$ after using $y(0) = -2$. The initial value is negative, so take the negative root: **$y = -\sqrt{(x^2 + 4)}$**.

6. Separate: $dy/(1 + y^2) = 2x\,dx$. Integrate: $\tan^{-1} y = x^2 + C$. $y(0) = 0$ gives $C = 0$. **$y = \tan(x^2)$**.