Euler's Method and Logistic Growth

Differential Equations (BC) · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$y_{n+1} = y_n + h \cdot f(x_n, y_n)$ $dP/dt = kP(1 - P/L)$ $P(t) = L/(1 + Ae^{-kt})$, $A = (L - P_0)/P_0$

$\lim_{t \to \infty} P(t) = L$

Questions

Show your working in the space under each question.

1. EASY Use Euler's method with step 1 to estimate y(2) for dy/dx = 2x, y(0) = 0.

2. EASY For dP/dt = 0.3P(1 − P/1000), find the limit of P as t → ∞ and say what happens if P = 1200.

3. MEDIUM Use Euler's method with step 0.25 to estimate y(1) for dy/dx = y, y(0) = 1.

4. MEDIUM For dP/dt = 0.04P(200 − P), find the carrying capacity and the maximum growth rate.

5. HARD Use Euler's method with step 0.5 to estimate y(1) for dy/dx = x − y, y(0) = 2.

6. HARD Solve dP/dt = 0.5P(1 − P/80) with P(0) = 20 and find P(4).

Step-by-step answers

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1. Step 1: slope 0, y stays 0 at x = 1. Step 2: slope 2, y = 0 + 1(2) = **2**. (Exact: $y = x^2 = 4$.)

2. The carrying capacity is **1000**. At P = 1200 the bracket is negative, so dP/dt < 0 and the population **decreases toward 1000**.

3. Each step multiplies y by (1 + 0.25). After 4 steps: $1.25^4 \approx$ **2.44**. (Exact: e ≈ 2.718; Euler underestimates because the curve is concave up.)

4. dP/dt = 0 at P = 200, the carrying capacity. Growth is fastest at P = 100: 0.04(100)(100) = **400 per unit time**.

5. At (0, 2): slope −2, y = 2 + 0.5(−2) = 1. At (0.5, 1): slope −0.5, y = 1 + 0.5(−0.5) = **0.75**.

6. A = (80 − 20)/20 = 3, so $P = 80/(1 + 3e^{-0.5t})$. $P(4) = 80/(1 + 3e^{-2}) = 80/1.406 \approx$ **56.9**.