Implicit Differentiation and Inverse Functions

Composite, Implicit and Inverse Functions · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$d/dx\,[y^n] = ny^{n-1} \cdot dy/dx$ $d/dx\,[xy] = y + x \cdot dy/dx$ $(f^{-1})'(b) = 1/f'(a)$, where $f(a) = b$

$d^2y/dx^2 = d/dx\,(dy/dx)$

Questions

Show your working in the space under each question.

1. EASY Find dy/dx if $y^3 = x^2$.

2. EASY Find the slope of $x^2 - y^2 = 16$ at $(5, 3)$.

3. MEDIUM Use implicit differentiation on $\sin y = x$ to find dy/dx in terms of x (for $-\pi/2 < y < \pi/2$).

4. MEDIUM Find the tangent line to $xy = 12$ at $(3, 4)$.

5. HARD $f(x) = x^3 + x + 1$. Find $(f^{-1})'(3)$.

6. HARD Find the slope of the folium $x^3 + y^3 = 6xy$ at (3, 3).

Step-by-step answers

Implicit Differentiation and Inverse Functions

1. $3y^2 \cdot dy/dx = 2x$, so $\mathbf{dy/dx = 2x/(3y^2)}$.

2. $2x - 2y \cdot dy/dx = 0$, so $dy/dx = x/y = \mathbf{5/3}$.

3. $\cos y \cdot dy/dx = 1$, so $dy/dx = 1/\cos y$. Since $\cos y = \sqrt{(1 - \sin^2 y)} = \sqrt{(1 - x^2)}$, $\mathbf{dy/dx = 1/\sqrt{(1 - x^2)}}$. This is the derivative of $\sin^{-1} x$.

4. Product rule: $y + x \cdot dy/dx = 0$, so $dy/dx = -y/x = -4/3$. Line: $\mathbf{y - 4 = -(4/3)(x - 3)}$.

5. Find a with $f(a) = 3$: $f(1) = 1 + 1 + 1 = 3$, so $a = 1$. $f'(x) = 3x^2 + 1$, $f'(1) = 4$. So $(f^{-1})'(3) = \mathbf{1/4}$.

6. $3x^2 + 3y^2 y' = 6y + 6x y'$. So $y'(3y^2 - 6x) = 6y - 3x^2$ and $y' = (6y - 3x^2)/(3y^2 - 6x)$. At $(3, 3)$: $(18 - 27)/(27 - 18) = \mathbf{-1}$.