Improper Integrals

Integration and Accumulation of Change (BC) · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$ $\int_1^\infty dx/x^p = 1/(p-1)$ if $p > 1$ $\int_0^1 dx/x^p = 1/(1-p)$ if $p < 1$ $\int_0^\infty e^{-kx}\,dx = 1/k$

Questions

Show your working in the space under each question.

1. EASY Evaluate $\int_1^\infty dx/x^3$.

2. EASY Does $\int_1^\infty dx/\sqrt{x}$ converge?

3. MEDIUM Evaluate $\int_0^1 x^{-1/3}\,dx$.

4. MEDIUM Evaluate $\int_{-\infty}^0 e^{3x}\,dx$.

5. HARD Evaluate $\int_0^\infty dx/(1 + x^2)$.

6. HARD Evaluate $\int_2^\infty dx/(x^2 - 1)$.

Step-by-step answers

Improper Integrals

1. $[-1/(2x^2)]$ from 1 to b = $1/2 - 1/(2b^2) \to$ **1/2**.

2. $p = 1/2 \le 1$. $\int_1^b x^{-1/2}\,dx = 2\sqrt{b} - 2 \to \infty$. It **diverges**.

3. The integrand blows up at 0. $\int_a^1 x^{-1/3}\,dx = (3/2)(1 - a^{2/3}) \to$ **3/2** as $a \to 0$.

4. $[e^{3x}/3]$ from a to 0 = $1/3 - e^{3a}/3 \to$ **1/3** as $a \to -\infty$.

5. $[\tan^{-1} x]$ from 0 to b = $\tan^{-1} b \to$ **π/2**.

6. Partial fractions: antiderivative $(1/2)\ln[(x-1)/(x+1)]$. As $x \to \infty$ the fraction $\to 1$ and the log $\to 0$. At $x = 2$ it is $(1/2)\ln(1/3)$. Result: $0 - (1/2)\ln(1/3) =$ **$(1/2)\ln 3 \approx 0.549$**.