Accumulation, Net Change and Motion with Integrals

Applications of Integration · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$F(b) = F(a) + \int_a^b F'(t)\,dt$ $\quad$ displacement $= \int_a^b v(t)\,dt$ $\quad$ distance $= \int_a^b |v(t)|\,dt$

$v(t) = v(0) + \int_0^t a(u)\,du$

Questions

Show your working in the space under each question.

1. EASY $a(t) = 6t$ and $v(0) = 2$. Find $v(3)$.

2. EASY $f(2) = 5$ and $\int_2^6 f'(x)\,dx = 7$. Find $f(6)$.

3. MEDIUM A population grows at $200e^{0.1t}$ people per year. By how much does it grow in the first 10 years?

4. MEDIUM $v(t) = 6 - 2t$ for $0 \le t \le 5$. Find the displacement and the distance.

5. HARD $a(t) = 6t - 6$, $v(0) = 0$ and $s(0) = 1$. Find the total distance travelled on $0 \leq t \leq 3$.

6. HARD $f(t) = 2$ for $0 \leq t \leq 2$, then falls in a straight line to -2 at $t = 4$. Let $g(x) = \int_0^x f(t)\,dt$. Find the maximum of g on $[0, 4]$.

Step-by-step answers

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1. $v(3) = 2 + \int_0^3 6t\,dt = 2 + [3t^2] = 2 + 27 = \mathbf{29}$.

2. Net change: $f(6) = f(2) + 7 = \mathbf{12}$.

3. $\int_0^{10} 200e^{0.1t}\,dt = [2000e^{0.1t}] = 2000(e - 1) \approx \mathbf{3437\ people}$.

4. Displacement: $[6t - t^2]$ from 0 to 5 $= 30 - 25 = \mathbf{5}$. $v = 0$ at $t = 3$: from 0 to 3 the particle moves $+9$, from 3 to 5 it moves -4. Distance $= 9 + 4 = \mathbf{13}$.

5. $v = 3t^2 - 6t$ and $s = t^3 - 3t^2 + 1$. $v = 3t(t - 2)$ changes sign at $t = 2$. $s(0) = 1$, $s(2) = -3$, $s(3) = 1$. Distance $= 4 + 4 = \mathbf{8}$.

6. $g' = f$, which is positive until the line crosses zero at $t = 3$, then negative. So g peaks at $x = 3$. $g(3)$ = rectangle $2 \times 2 = 4$ plus triangle $(1/2)(1)(2) = 1$, giving $\mathbf{g(3) = 5}$.