L'Hôpital's Rule

Contextual Applications of Differentiation · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\lim f/g = \lim f'/g'$ $0 \cdot \infty \to$ rewrite as $f/(1/g)$ $y = f^g \Rightarrow \ln y = g \ln f$ $\lim_{x\to\infty} (1 + a/x)^x = e^a$

Questions

Show your working in the space under each question.

1. EASY Use L'Hôpital's rule to find $\lim_{x\to 2} (x^2 - 4)/(x - 2)$.

2. EASY Find $\lim_{x\to 0} \sin(4x)/\sin(3x)$.

3. MEDIUM Find $\lim_{x\to 0} (1 - \cos x)/x^2$.

4. MEDIUM Find $\lim_{x\to\infty} (\ln x)/\sqrt{x}$.

5. HARD Find $\lim_{x\to 0^+} x \ln x$.

6. HARD Find $\lim_{x \to 0} (e^x - 1 - x)/x^2$.

Step-by-step answers

L'Hôpital's Rule

1. 0/0. Differentiate: $2x/1 \to$ **4** (the same answer as factoring).
2. 0/0. Differentiate: $4 \cos 4x/(3 \cos 3x) \to$ **4/3**.
3. 0/0. First pass: $\sin x/(2x)$, still 0/0. Second pass: $\cos x/2 \to$ **1/2**.
4. ∞/∞. Differentiate: $(1/x)/(1/(2\sqrt{x})) = 2/\sqrt{x} \to$ **0**. Logs grow more slowly than any root.
5. Form $0 \cdot (-\infty)$. Rewrite as $\ln x/(1/x)$, which is ∞/∞. Differentiate: $(1/x)/(-1/x^2) = -x \to$ **0**.
6. 0/0. First pass: $(e^x - 1)/(2x)$, still 0/0. Second pass: $e^x/2 \to$ **1/2**.