# Limits and Limit Laws

Limits and Continuity · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

**USEFUL EQUATIONS**

$\lim [f(x) \pm g(x)] = \lim f \pm \lim g$ $\lim_{x\to 0} (\sin x)/x = 1$ $\lim_{x\to 0} (1 - \cos x)/x = 0$

$\lim_{x\to \infty} 1/x^n = 0$

## Questions

Show your working in the space under each question.

**1.** EASY Find $\lim_{x\to 2} (3x^2 - 4x + 1)$.

**2.** EASY Find $\lim_{x\to -1} (x^2 + 3x + 2)/(x + 1)$.

**3.** MEDIUM Find $\lim_{x\to 0} \sin(3x)/(4x)$.

**4.** MEDIUM Find $\lim_{x\to \infty} (5x^3 - 2x)/(2x^3 + 7)$.

**5.** HARD Find $\lim_{x \to 1} (1/x - 1)/(x - 1)$.

**6.** HARD Find $\lim_{x \to 0} (1 - \cos x)/x^2$.

## Step-by-step answers

Limits and Limit Laws

1. A polynomial is continuous, so substitute: $3(4) - 4(2) + 1 = 12 - 8 + 1 = \mathbf{5}$.
2. Substituting gives 0/0. Factor: $(x + 1)(x + 2)/(x + 1) = x + 2$. Substitute $x = -1$: $\mathbf{1}$.
3. Rewrite as $(3/4) \cdot \sin(3x)/(3x)$. As $x \to 0$, $\sin(3x)/(3x) \to 1$, so the limit is $\mathbf{3/4}$.
4. Divide top and bottom by $x^3$: $(5 - 2/x^2)/(2 + 7/x^3)$. The small terms go to 0, leaving $\mathbf{5/2}$.
5. Combine the top: $1/x - 1 = (1 - x)/x$. Then $(1 - x)/[x(x - 1)] = -1/x$. Substitute $x = 1$: $\mathbf{-1}$.
6. Multiply top and bottom by $(1 + \cos x)$: the top becomes $1 - \cos^2 x = \sin^2 x$. So the expression is $(\sin x / x)^2 \cdot 1/(1 + \cos x)$. As $x \to 0$ this is $1^2 \cdot 1/2 = \mathbf{1/2}$.