Motion Along a Line

Contextual Applications of Differentiation · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$v = ds/dt$ $a = dv/dt = d^2s/dt^2$ speed $= |v|$ displacement $= s(b) - s(a)$

Questions

Show your working in the space under each question.

1. EASY $s(t) = 5t^2 + 2t$ (m). Find the velocity at $t = 3$ s.

2. EASY $s(t) = t^2 - 8t + 3$. When is the particle at rest and where is it then?

3. MEDIUM A ball has height $h(t) = 20t - 5t^2$ (m). Find its maximum height and its speed when it lands.

4. MEDIUM $s(t) = t^3 - 3t$ for $t \geq 0$. Find the acceleration at the moment the velocity is zero.

5. HARD $s(t) = t^2 - 4t$. Find the total distance travelled on $0 \leq t \leq 5$.

6. HARD $s(t) = t^4 - 4t^3$ for $t > 0$. When is the particle speeding up?

Step-by-step answers

Motion Along a Line

1. $v = s' = 10t + 2$, so $v(3) =$ **32 m/s**.

2. $v = 2t - 8 = 0$ gives $t =$ **4**. $s(4) = 16 - 32 + 3 =$ **−13**.

3. $v = 20 - 10t = 0$ at $t = 2$ s, so $h(2) = 40 - 20 =$ **20 m**. It lands when $h = 0$: $5t(4 - t) = 0$, $t = 4$ s. $v(4) = -20$, so the speed is **20 m/s**.

4. $v = 3t^2 - 3 = 0$ gives $t = 1$. $a = 6t$, so $a(1) =$ **6**.

5. $v = 2t - 4 = 0$ at $t = 2$. $s(0) = 0$, $s(2) = -4$, $s(5) = 5$. Distance $= |-4 - 0| + |5 - (-4)| = 4 + 9 =$ **13**.

6. $v = 4t^3 - 12t^2 = 4t^2(t - 3)$: negative for $0 < t < 3$, positive for $t > 3$. $a = 12t^2 - 24t = 12t(t - 2)$: negative for $0 < t < 2$, positive for $t > 2$. Same signs on **$0 < t < 2$** (both negative) and **$t > 3$** (both positive).