Optimisation

Analytical Applications of Differentiation · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$Q = f(x)$ $f'(x) = 0$ $f''(x) < 0 \Rightarrow$ maximum $f''(x) > 0 \Rightarrow$ minimum

Questions

Show your working in the space under each question.

1. EASY A rectangle has perimeter 40 m. What is its largest possible area?

2. EASY A farmer has 120 m of fence for three sides of a rectangular field along a river. Find the largest area.

3. MEDIUM Profit is $P(x) = -2x^2 + 80x - 300$. Find the maximum profit.

4. MEDIUM A rectangle sits on the x-axis with its top corners on $y = 12 - x^2$. Find its largest area.

5. HARD An open-top box with a square base must hold 32 m^3. Find the dimensions that use the least material.

6. HARD Find the points on $y = x^2$ closest to (0, 3).

Step-by-step answers

Optimisation

1. Sides x and $20 - x$. $A = 20x - x^2$, $A' = 20 - 2x = 0$ at $x = 10$. **$A = 100$ m²** (a square).

2. Two sides of length x and one of $120 - 2x$. $A = 120x - 2x^2$, $A' = 120 - 4x = 0$ at $x = 30$. Dimensions 30 m × 60 m, **$A = 1800$ m²**.

3. $P' = -4x + 80 = 0$ at $x = 20$. $P'' = -4 < 0$. $P(20) = -800 + 1600 - 300 =$ **500**.

4. Corners at $\pm x$, so width $2x$ and height $12 - x^2$. $A = 24x - 2x^3$, $A' = 24 - 6x^2 = 0$ at $x = 2$. **$A = 4 \times 8 = 32$**.

5. Base x by x, height $h = 32/x^2$. $S = x^2 + 4xh = x^2 + 128/x$. $S' = 2x - 128/x^2 = 0$ gives $x^3 = 64$, $x = 4$. **4 m × 4 m × 2 m**, with $S = 48$ m².

6. Minimise $D^2 = x^2 + (x^2 - 3)^2$. Derivative: $2x + 2(x^2 - 3)(2x) = 2x(2x^2 - 5) = 0$. $x = 0$ gives $D = 3$; $x^2 = 5/2$ gives $D^2 = 2.5 + 0.25 = 2.75$, $D \approx 1.66$. Closest points: **(±1.58, 2.5)**.