Parametric Equations and Vector-Valued Functions

Parametric Equations, Polar Coordinates and Vector-Valued Functions · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$dy/dx = (dy/dt)/(dx/dt)$ $d^2y/dx^2 = [d/dt\ (dy/dx)]/(dx/dt)$ speed $= \sqrt{((dx/dt)^2 + (dy/dt)^2)}$

$L = \int_a^b \sqrt{((dx/dt)^2 + (dy/dt)^2)}\ dt$

Questions

Show your working in the space under each question.

1. EASY Describe the curve $x = 3\cos t$, $y = 3\sin t$ and find the point at $t = \pi/2$.

2. EASY For $x = t + 1$, $y = t^2 - 1$, find dy/dx at $t = 2$.

3. MEDIUM For $x = t^2$, $y = t^3$, find d^2y/dx^2.

4. MEDIUM A particle has position (t^3, t^2). Find its speed and acceleration vector at $t = 2$.

5. HARD Find the length of the spiral $x = e^t \cos t$, $y = e^t \sin t$ from $t = 0$ to $t = 1$.

6. HARD A particle starts at $(1, 0)$ with velocity $(2t - 4, 3t^2)$. Find its position at $t = 2$ and describe its motion at that instant.

Step-by-step answers

Parametric Equations and Vector-Valued Functions

1. $x^2 + y^2 = 9$: a **circle of radius 3**, traced anticlockwise. At $t = \pi/2$ the point is **(0, 3)**.

2. $dy/dt = 2t$, $dx/dt = 1$, so $dy/dx = 2t = \mathbf{4}$.

3. $dy/dx = 3t^2/(2t) = 3t/2$. Differentiate with respect to t: 3/2. Divide by $dx/dt = 2t$: **3/(4t)**.

4. Velocity $(3t^2, 2t) = (12, 4)$, so speed $= \sqrt{(144 + 16)} = \sqrt{160} \approx \mathbf{12.65}$. Acceleration $(6t, 2) = \mathbf{(12, 2)}$.

5. $x' = e^t(\cos t - \sin t)$, $y' = e^t(\sin t + \cos t)$. $x'^2 + y'^2 = 2e^{2t}$, so speed $= \sqrt{2}\, e^t$. $L = \sqrt{2}(e - 1) \approx \mathbf{2.43}$.

6. $x = 1 + \int_0^2 (2t - 4)\, dt = 1 + (4 - 8) = -3$. $y = 0 + \int_0^2 3t^2\, dt = 8$. Position **(−3, 8)**. At $t = 2$ the velocity is $(0, 12)$: it is moving straight up, so the path has a vertical tangent.