Integration by Parts and Partial Fractions

Integration and Accumulation of Change (BC) · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\int u\,dv = uv - \int v\,du \quad \int_a^b u\,dv = [uv]_a^b - \int_a^b v\,du$

$(px+q)/[(x-a)(x-b)] = A/(x-a) + B/(x-b) \quad \int dx/(x-a) = \ln|x-a| + C$

Questions

Show your working in the space under each question.

1. EASY Find $\int xe^{2x}\,dx$.

2. EASY Find $\int \ln x\,dx$.

3. MEDIUM Evaluate $\int_0^1 xe^{-x}\,dx$.

4. MEDIUM Find $\int dx/(x^2-1)$.

5. HARD Find $\int x^2 e^x \, dx$.

6. HARD Find $\int e^x \sin x \, dx$.

Step-by-step answers

Integration by Parts and Partial Fractions

1. u = x, dv = e^{2x} dx, v = $e^{2x}/2$. Result: $\mathbf{xe^{2x}/2 - e^{2x}/4 + C}$.

2. u = ln x, dv = dx, v = x. x ln x − ∫ x(1/x) dx = **x ln x − x + C**.

3. u = x, v = $-e^{-x}$. $[-xe^{-x}]$ from 0 to 1 + $\int_0^1 e^{-x}\,dx = -1/e + (1 - 1/e) = \mathbf{1 - 2/e \approx 0.264}$.

4. 1/[(x − 1)(x + 1)] = A/(x − 1) + B/(x + 1) with A = 1/2, B = −1/2. Result: **(1/2) ln|(x − 1)/(x + 1)| + C**.

5. Parts twice. First: $x^2e^x - \int 2xe^x\,dx$. Second: $\int 2xe^x\,dx = 2xe^x - 2e^x$. Result: $\mathbf{e^x(x^2 - 2x + 2) + C}$.

6. Let I be the integral. Parts twice (u = sin x, then u = cos x): $I = e^x \sin x - e^x \cos x - I$. So $2I = e^x(\sin x - \cos x)$ and $\mathbf{I = e^x(\sin x - \cos x)/2 + C}$.