Polar Curves

Parametric Equations, Polar Coordinates and Vector-Valued Functions · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2 + y^2 \qquad A = (1/2)\int_\alpha^\beta r^2\,d\theta$

$dy/dx = (r'\sin\theta + r\cos\theta)/(r'\cos\theta - r\sin\theta) \qquad L = \int_\alpha^\beta \sqrt{(r^2 + (r')^2)}\,d\theta$

Questions

Show your working in the space under each question.

1. EASY Convert the Cartesian point $(-1, 1)$ to polar coordinates.

2. EASY Use the polar area formula to find the area of the circle $r = 5$.

3. MEDIUM Find the area enclosed by the cardioid $r = 1 + \cos\theta$.

4. MEDIUM Find the slope of $r = 1 + \cos\theta$ at $\theta = \pi/2$.

5. HARD Find the area of one petal of $r = \cos 3\theta$.

6. HARD Find the total length of the cardioid $r = 1 + \cos \theta$.

Step-by-step answers

Polar Curves

1. $r = \sqrt{(1 + 1)} = \mathbf{\sqrt{2}}$. The point is in the second quadrant with $\tan\theta = -1$, so $\mathbf{\theta = 3\pi/4}$.

2. $A = (1/2)\int_0^{2\pi} 25\, d\theta = (1/2)(25)(2\pi) = \mathbf{25\pi}$.

3. $A = (1/2)\int_0^{2\pi} (1 + 2\cos\theta + \cos^2\theta)\, d\theta = (1/2)(2\pi + 0 + \pi) = \mathbf{3\pi/2}$.

4. $r = 1$ and $r' = -\sin\theta = -1$. $dy/dx = (r'\sin\theta + r\cos\theta)/(r'\cos\theta - r\sin\theta) = (-1 + 0)/(0 - 1) = \mathbf{1}$.

5. One petal runs from $\theta = -\pi/6$ to $\pi/6$. $A = (1/2)\int \cos^2 3\theta\, d\theta = (1/2)(\pi/6) = \mathbf{\pi/12}$.

6. $r^2 + (r')^2 = (1 + \cos\theta)^2 + \sin^2\theta = 2 + 2\cos\theta = 4\cos^2(\theta/2)$. $L = \int_0^{2\pi} 2|\cos(\theta/2)|\, d\theta = 2 \cdot \int_0^{\pi} 2\cos(\theta/2)\, d\theta = 2 \cdot 4 = \mathbf{8}$.