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Contextual Applications of Differentiation · 6 questions · step-by-step answers at the end

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USEFUL EQUATIONS

$A = \pi r^2 \Rightarrow dA/dt = 2\pi r \cdot dr/dt$ $V = (4/3)\pi r^3 \Rightarrow dV/dt = 4\pi r^2 \cdot dr/dt$

$x^2 + y^2 = L^2 \Rightarrow x \cdot dx/dt + y \cdot dy/dt = 0$ $V = (1/3)\pi r^2 h$

Questions

Show your working in the space under each question.

1. EASY The side of a cube grows at 0.5 cm/s. How fast is its volume growing when the side is 4 cm?

2. EASY A spherical balloon is inflated at 100 cm^3/s. How fast is the radius growing when r = 5 cm?

3. MEDIUM Car A is 30 km east of a junction driving east at 60 km/h. Car B is 40 km north of it driving north at 80 km/h. How fast is the distance between them growing?

4. MEDIUM A 1.8 m tall person walks away from a 6 m lamp post at 1.2 m/s. How fast is their shadow lengthening?

5. HARD A balloon rises vertically at 3 m/s from a point 40 m from an observer. How fast is the angle of elevation increasing when the balloon is 30 m high?

6. HARD A point moves along $y = x^2$ with $dx/dt = 3$ units/s. How fast is its distance from the origin changing when $x = 1$?

Step-by-step answers

Related Rates

1. $V = s^3$, so $dV/dt = 3s^2 \cdot ds/dt = 3(16)(0.5) = \mathbf{24\ cm^3/s}$.

2. $dV/dt = 4\pi r^2 \cdot dr/dt$. So $100 = 4\pi(25) \cdot dr/dt$ and $dr/dt = 1/\pi \approx \mathbf{0.318\ cm/s}$.

3. $D^2 = x^2 + y^2$, so $D \cdot dD/dt = x \cdot dx/dt + y \cdot dy/dt$. $D = 50$. $dD/dt = (30 \cdot 60 + 40 \cdot 80)/50 = 5000/50 = \mathbf{100\ km/h}$.

4. Let x = distance from the post and s = shadow length. Similar triangles: $s/1.8 = (x + s)/6$, so $6s = 1.8x + 1.8s$ and $s = (3/7)x$. Then $ds/dt = (3/7)(1.2) \approx \mathbf{0.514\ m/s}$.

5. $\tan\theta = h/40$. Differentiate: $\sec^2\theta \cdot d\theta/dt = (1/40) \cdot dh/dt$. At $h = 30$, $\tan\theta = 0.75$, so $\sec^2\theta = 1 + 0.5625 = 1.5625$. $d\theta/dt = (3/40)/1.5625 = \mathbf{0.048\ rad/s}$.

6. $D^2 = x^2 + y^2 = x^2 + x^4$. Differentiate: $2D \cdot dD/dt = (2x + 4x^3) \cdot dx/dt$. At $x = 1$: $D = \sqrt{2}$, so $2\sqrt{2} \cdot dD/dt = 6 \cdot 3 = 18$ and $dD/dt = 9/\sqrt{2} \approx \mathbf{6.36\ units/s}$.