Riemann Sums, Definite Integrals and the Fundamental Theorem

Integration and Accumulation of Change · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\int_a^b f(x)\,dx = F(b) - F(a)$ $\quad \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ $\quad \frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x))\cdot g'(x)$

$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$

Questions

Show your working in the space under each question.

1. EASY Evaluate $\int_0^2 3x^2\,dx$.

2. EASY Evaluate $\int_0^{\pi} \sin x\,dx$.

3. MEDIUM Use a midpoint sum with 2 strips to estimate $\int_0^4 x^2\,dx$.

4. MEDIUM A table gives $f(0) = 3$, $f(2) = 5$, $f(4) = 4$, $f(6) = 8$. Estimate $\int_0^6 f(x)\,dx$ with three trapezoids.

5. HARD Find $\frac{d}{dx} \int_2^{x^3} \cos(t^2)\, dt$.

6. HARD Find the average value of $f(x) = x^2$ on $[0, 3]$ and the value of c where $f(c)$ equals it.

Step-by-step answers

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1. $[x^3]$ from 0 to 2 = **8**.

2. $[-\cos x]$ from 0 to $\pi = -(-1) - (-1) =$ **2**.

3. $\Delta x = 2$, midpoints 1 and 3. Sum = $2(1 + 9) =$ **20** (exact $64/3 \approx 21.3$).

4. Each width is 2. Sum = $(2/2)[3 + 2(5) + 2(4) + 8] =$ **29**.

5. FTC part 1 with the chain rule: $\cos((x^3)^2) \cdot 3x^2 =$ **$3x^2 \cos(x^6)$**.

6. Average = $(1/3)\int_0^3 x^2\,dx = (1/3)(9) =$ **3**. $f(c) = c^2 = 3$ gives **$c = \sqrt{3} \approx 1.73$**.