Infinite Series and Convergence Tests

Infinite Sequences and Series · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$\Sigma_{n=0}^{\infty} ar^n = a/(1 - r),\ |r| < 1 \quad \Sigma\ 1/n^p$ converges $\Leftrightarrow p > 1 \quad L = \lim |a_{n+1}/a_n|$

$|S - S_n| \le |a_{n+1}|$

Questions

Show your working in the space under each question.

1. EASY Find the sum of $\Sigma\ 2/3^n$ from $n = 1$ to ∞.

2. EASY Does $\Sigma\ n/(2n + 1)$ converge?

3. MEDIUM Classify $\Sigma\ 1/n^{1.5}$ and $\Sigma\ 1/\sqrt{n}$.

4. MEDIUM Does $\Sigma\ n!/10^n$ converge?

5. HARD Use the integral test on $\Sigma\ 1/(n \ln n)$ from $n = 2$.

6. HARD How many terms of $\Sigma\ (-1)^{n+1}/n^2$ are needed for an error below 0.01?

Step-by-step answers

Infinite Series and Convergence Tests

1. Geometric with first term 2/3 and r = 1/3. Sum = (2/3)/(1 − 1/3) = **1**.
2. The terms tend to 1/2, not 0. By the nth-term test it **diverges**.
3. p-series: p = 1.5 > 1 **converges**; p = 0.5 ≤ 1 **diverges**.
4. Ratio: $[(n + 1)!/10^{n+1}] \cdot [10^n/n!] = (n + 1)/10 \to \infty$. Greater than 1, so it **diverges**.
5. $\int_2^\infty dx/(x \ln x)$: let u = ln x, giving ln(ln x), which grows without bound. The integral diverges, so the series **diverges**.
6. The error is at most the next term, $1/(n + 1)^2$. Need $1/(n + 1)^2 < 0.01$, so n + 1 > 10. **n = 10 terms** (then the bound is 1/121 ≈ 0.0083).