Tangent Lines, Linear Approximation and the Mean Value Theorem

Analytical Applications of Differentiation · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$L(x) = f(a) + f'(a)(x - a)$ $f'(c) = [f(b) - f(a)]/(b - a)$ $m_{normal} = -1/m_{tangent}$

$f(a) = f(b) \Rightarrow f'(c) = 0$

Questions

Show your working in the space under each question.

1. EASY Find the normal line to $y = x^2$ at (1, 1).

2. EASY Where is the tangent to $y = x^2 - 6x + 5$ horizontal?

3. MEDIUM Linearise $f(x) = \ln x$ at $x = 1$ and estimate $\ln 1.1$.

4. MEDIUM Verify Rolle's theorem for $f(x) = x^2 - 4x$ on $[0, 4]$ and find c.

5. HARD Find the c guaranteed by the MVT for $f(x) = \sqrt{x}$ on $[1, 9]$.

6. HARD Use a linear approximation to estimate the cube root of 8.06.

Step-by-step answers

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1. Tangent slope: $2x = 2$. Normal slope: $-1/2$. Line: $\mathbf{y - 1 = -(1/2)(x - 1)}$.
2. $y' = 2x - 6 = 0$ at $x = 3$. $y = 9 - 18 + 5 = -4$. The point is $\mathbf{(3, -4)}$.
3. $f(1) = 0$, $f'(1) = 1$, so $L(x) = x - 1$. $\ln 1.1 \approx \mathbf{0.1}$ (true value 0.0953).
4. $f(0) = 0$ and $f(4) = 0$, and f is a polynomial, so the theorem applies. $f'(c) = 2c - 4 = 0$ gives $\mathbf{c = 2}$.
5. Average rate: $(3 - 1)/(9 - 1) = 1/4$. $f'(c) = 1/(2\sqrt{c}) = 1/4$ gives $\sqrt{c} = 2$, so $\mathbf{c = 4}$.
6. $f(x) = x^{1/3}$, $a = 8$. $f(8) = 2$, $f'(x) = (1/3)x^{-2/3}$, $f'(8) = 1/12$. $L(8.06) = 2 + 0.06/12 = \mathbf{2.005}$.