Derivatives of Trig, Exponential and Log Functions

Differentiation: Definition and Basic Rules · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$ $(e^x)' = e^x$, $(a^x)' = a^x \ln a$

$(\ln x)' = 1/x$, $(\log_a x)' = 1/(x \ln a)$ $(\sin^{-1} x)' = 1/\sqrt{(1 - x^2)}$, $(\tan^{-1} x)' = 1/(1 + x^2)$

Questions

Show your working in the space under each question.

1. EASY Differentiate $y = \tan x + \ln x$.

2. EASY Differentiate $y = 5^x + e^{-3x}$.

3. MEDIUM Differentiate $y = \ln(x^2 + 1)$.

4. MEDIUM Differentiate $y = \sin^2(4x)$.

5. HARD Differentiate $y = \tan^{-1}(2x)$.

6. HARD Find the slope of $y = e^x/(1 + e^x)$ at $x = 0$.

Step-by-step answers

Derivatives of Trig, Exponential and Log Functions

1. $\mathbf{y' = \sec^2 x + 1/x}$.
2. $(5^x)' = 5^x \ln 5$ and $(e^{-3x})' = -3e^{-3x}$. So $\mathbf{y' = 5^x \ln 5 - 3e^{-3x}}$.
3. Inside $g = x^2 + 1$, $g' = 2x$. $y' = g'/g = \mathbf{2x/(x^2 + 1)}$.
4. Three layers: square, sine, 4x. $y' = 2 \sin(4x) \cdot \cos(4x) \cdot 4 = 8 \sin 4x \cos 4x$, which equals $\mathbf{4 \sin 8x}$.
5. $y' = 1/(1 + (2x)^2) \cdot 2 = \mathbf{2/(1 + 4x^2)}$.
6. Quotient rule: $y' = [e^x(1 + e^x) - e^x \cdot e^x]/(1 + e^x)^2 = e^x/(1 + e^x)^2$. At $x = 0$: $1/(2)^2 = \mathbf{1/4}$.