# Volumes of Solids

Applications of Integration · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

**USEFUL EQUATIONS**

$V = \pi \int_a^b R^2\,dx$ $V = \pi \int_a^b (R^2 - r^2)\,dx$ $V = \int_a^b A(x)\,dx$ $V = 2\pi \int_a^b (\text{radius})(\text{height})\,dx$

## Questions

Show your working in the space under each question.

**1.** EASY Rotate the region under $y = \sqrt{x}$ from 0 to 4 about the x-axis. Find the volume.

**2.** EASY Rotate the region under $y = x^2$ from 0 to 1 about the x-axis. Find the volume.

**3.** MEDIUM The region bounded by $y = x^2$, $y = 4$ and the y-axis (first quadrant) is rotated about the y-axis. Find the volume.

**4.** MEDIUM The region between $y = \sqrt{x}$ and $y = x/2$ is rotated about the x-axis. Find the volume.

**5.** HARD The base of a solid is the circle $x^2 + y^2 = 4$. Cross-sections perpendicular to the x-axis are squares. Find the volume.

**6.** HARD Use shells to find the volume when the region under $y = x^2$ from 0 to 2 is rotated about the y-axis.

## Step-by-step answers

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**1.** $V = \pi \int_0^4 x\,dx = \pi[x^2/2] = \mathbf{8\pi}$.

**2.** $V = \pi \int_0^1 x^4\,dx = \mathbf{\pi/5}$.

**3.** Horizontal disks of radius $x = \sqrt{y}$: $V = \pi \int_0^4 y\,dy = \pi[y^2/2] = \mathbf{8\pi}$.

**4.** They meet at $x = 0$ and 4, with $\sqrt{x}$ on top. $V = \pi \int_0^4 (x - x^2/4)\,dx = \pi[x^2/2 - x^3/12] = \pi(8 - 16/3) = \mathbf{8\pi/3}$.

**5.** Side of the square $= 2\sqrt{(4 - x^2)}$, so $A(x) = 4(4 - x^2)$. $V = \int_{-2}^2 4(4 - x^2)\,dx = 4(32/3) = \mathbf{128/3}$.

**6.** Shell radius $x$, height $x^2$: $V = 2\pi \int_0^2 x^3\,dx = 2\pi(4) = \mathbf{8\pi}$.