Gravitation, Orbits & Kepler's Laws

Gravitation · 7 questions · step-by-step answers at the end

Name: Date: Score: / 7

USEFUL EQUATIONS

$F = GMm/r^2$ $v = \sqrt{(GM/r)}$ $T^2 = 4\pi^2 r^3/(GM)$ $U = -GMm/r$ $v_{esc} = \sqrt{(2GM/r)}$

Questions

Show your working in the space under each question.

1. A planet orbits the Sun with a semi-major axis of 4.0 AU. What is its period?

2. A satellite orbits 400 km above Earth's surface ($R_E = 6.37 \times 10^6$ m, $GM_E = 3.99 \times 10^{14}$ m³/s²). Find its speed and period.

3. The distance between two masses triples. What happens to the gravitational force?

4. What is g halfway to the centre of a uniform Earth?

5. Earth pulls on the Moon with force F. How hard does the Moon pull on Earth?

6. By what factor must a satellite's orbital radius increase to double its period?

7. Planet X has twice Earth's mass and twice its radius. What is g there?

Step-by-step answers

Gravitation, Orbits & Kepler's Laws

1. In years and AU, $T^2 = a^3$, so $T = \sqrt{(4^3)} =$ **8.0 years**.

2. 1. Orbital radius $r = 6.37 \times 10^6 + 0.40 \times 10^6 = 6.77 \times 10^6$ m.
2. $v = \sqrt{(GM/r)} = \sqrt{(3.99 \times 10^{14} / 6.77 \times 10^6)} =$ **7.67 km/s**.
3. $T = 2\pi r/v = 2\pi(6.77 \times 10^6)/7670 = 5540$ s $\approx$ **92 minutes**.
4. Compare escape speed from the surface: $\sqrt{(2GM/R)} \approx 11.2$ km/s.

Answer: v ≈ 7.7 km/s, T ≈ 92 min

3. It becomes **1/9** as large.

4. $g \propto r$ inside, so **4.9 m/s²**.

5. Also **F** (a Newton's 3rd law pair). Earth's acceleration is just much smaller.

6. $T \propto r^{3/2} \rightarrow r \propto T^{2/3}$: a factor of $2^{2/3} \approx$ **1.59**.

7. $g \propto M/R^2 = 2/4$, so **4.9 m/s²**.