Conic Sections

Pre-Calculus · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

$x^2/a^2 + y^2/b^2 = 1,\ c^2 = a^2 - b^2$ $x^2/a^2 - y^2/b^2 = 1,\ c^2 = a^2 + b^2$ $y = \pm(b/a)x$ $y^2 = 4px$

Questions

Show your working in the space under each question.

1. EASY Name each curve: $x^2 + y^2 = 9$; $y = x^2$; $x^2/4 + y^2 = 1$; $x^2 - y^2 = 1$.

2. EASY Find the focus and directrix of $x^2 = 8y$.

3. MEDIUM Find the vertices and foci of $x^2/16 + y^2/25 = 1$.

4. MEDIUM Write the equation of the ellipse with vertices $(\pm6, 0)$ and foci $(\pm4, 0)$.

5. HARD Find the vertices, foci and asymptotes of $y^2/4 - x^2/9 = 1$.

6. HARD Write $4x^2 + 9y^2 - 16x + 18y - 11 = 0$ in standard form and describe the curve.

Step-by-step answers

Conic Sections

1. **Circle, parabola, ellipse, hyperbola**.
2. $4p = 8$, so $p = 2$, and the parabola opens upward. Focus **(0, 2)**, directrix **$y = -2$**.
3. The larger number is under y^2, so the long axis is vertical: vertices **(0, ±5)**. $c^2 = 25 - 16 = 9$, so foci **(0, ±3)**.
4. $a = 6$ and $c = 4$, so $b^2 = 36 - 16 = 20$. **$x^2/36 + y^2/20 = 1$**.
5. It opens up and down: vertices **(0, ±2)**. $c^2 = 4 + 9 = 13$, so foci **(0, ±√13)**. Asymptotes **$y = \pm(2/3)x$**.
6. $4(x^2 - 4x) + 9(y^2 + 2y) = 11$. Complete the squares: $4(x - 2)^2 - 16 + 9(y + 1)^2 - 9 = 11$, so $4(x - 2)^2 + 9(y + 1)^2 = 36$. **$(x - 2)^2/9 + (y + 1)^2/4 = 1$**: an ellipse with centre $(2, -1)$, $a = 3$ and $b = 2$.