Rational Functions and Asymptotes

Algebra 2 · 6 questions · step-by-step answers at the end

Name: Date: Score: / 6

USEFUL EQUATIONS

denominator = 0 (not cancelled) ⇒ vertical asymptote

equal degrees ⇒ y = a/b (leading coefficients)

top = (bottom)(quotient) + remainder ⇒ y = quotient a/b = c/d ⇒ ad = bc

Questions

Show your working in the space under each question.

1. EASY State the domain of $f(x) = 3/(x + 5)$.

2. EASY Find the asymptotes of $y = (3x^2 + 1)/(x^2 - 4)$.

3. MEDIUM Simplify $(x^2 - 9)/(x - 3)$ and describe its graph.

4. MEDIUM Solve $2/(x - 1) = 3/(x + 2)$.

5. HARD Solve $(x + 1)/(x - 2) > 0$.

6. HARD Find the slant asymptote of $y = (x^2 + 1)/(x - 1)$.

Step-by-step answers

Rational Functions and Asymptotes

1. The bottom cannot be zero: **all real x except x = −5**.
2. Vertical: **x = 2 and x = −2**. Horizontal: **y = 3**.
3. $(x - 3)(x + 3)/(x - 3) = x + 3$. The graph is the line **y = x + 3 with a hole at (3, 6)**.
4. Cross-multiply: $2(x + 2) = 3(x - 1)$, so $2x + 4 = 3x - 3$ and **x = 7**.
5. The sign changes at $x = -1$ and $x = 2$. Test: both factors have the same sign when **x < −1 or x > 2**.
6. Divide: $x^2 + 1 = (x - 1)(x + 1) + 2$, so $y = x + 1 + 2/(x - 1)$. The slant asymptote is **y = x + 1**.