Conservative Forces & Potential Energy Curves

Energy · 4 questions · step-by-step answers at the end

Name: Date: Score: / 4

USEFUL EQUATIONS

$F(x) = -\mathrm{d}U/\mathrm{d}x \quad \Delta U = -\int F\,\mathrm{d}x \quad E = K + U(x) \quad \mathrm{d}U/\mathrm{d}x = 0$

Questions

Show your working in the space under each question.

1. For U(x) = 4x² (J), what is the force at x = 0.50 m?

2. A particle moves in U(x) = 3x² − x³ (U in J, x in m). Find the equilibrium points and classify them. How much energy does a particle resting at x = 0 need to escape past the hump?

3. U(x) = 5x² − 2x (J). Where is the equilibrium, and is it stable?

4. What is the force at x = 1 m for U(x) = 2/x (J)?

Step-by-step answers

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1. $F = -dU/dx = -8x =$ **-4.0 N**, pointing back toward $x = 0$.

2. 1. $F = -dU/dx = -(6x - 3x^2) = 3x^2 - 6x$.
2. Equilibrium where $dU/dx = 6x - 3x^2 = 3x(2 - x) = 0$, so $x = 0$ and $x = 2$ m.
3. $d^2U/dx^2 = 6 - 6x$: at $x = 0$ it's $+6$ (**stable**), and at $x = 2$ it's -6 (**unstable**).
4. The hump height is $U(2) = 12 - 8 = 4$ J, so it needs more than **4 J** of kinetic energy.

Answer: x = 0 is stable, x = 2 m is unstable; needs more than 4 J

3. $U' = 10x - 2 = 0 \rightarrow x =$ **0.2 m**; $U'' = 10 > 0$, so it is **stable**.

4. $F = -dU/dx = 2/x^2 =$ **+2 N**, which is repulsive.