Simple Harmonic Motion

Oscillations · 8 questions · step-by-step answers at the end

Name: Date: Score: / 8

USEFUL EQUATIONS

$T = 2\pi\sqrt{(m/k)}$ $T = 2\pi\sqrt{(L/g)}$ $x(t) = A\cos(\omega t + \varphi)$ $v_{max} = A\omega,\ a_{max} = A\omega^2$ $E = \frac{1}{2}kA^2$

Questions

Show your working in the space under each question.

1. How long must a pendulum be to have a period of 2.0 s on Earth?

2. A 0.50 kg mass on a spring with k = 200 N/m oscillates with amplitude 0.10 m. Find the period, the maximum speed and the total energy.

3. The mass on a spring is quadrupled. What happens to the period?

4. A 1.5 kg mass hangs from a spring with k = 300 N/m. How far does the spring stretch at equilibrium, and what is the period?

5. The amplitude of a mass–spring system is doubled. What happens to the period, maximum speed and total energy?

6. Two identical springs (k each) are joined in series to a mass m. How does the period compare with one spring?

7. Where in the motion is the acceleration greatest?

8. For $x = 0.04\cos(5t)$, find v_{max} and a_{max}.

Step-by-step answers

Simple Harmonic Motion

1. $L = gT^2/(4\pi^2) = 9.8(4.0)/39.5 \approx$ **0.99 m**.

2. 1. $T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(0.50/200)} =$ **0.31 s**.
2. $\omega = \sqrt{(k/m)} = \sqrt{400} = 20$ rad/s, so $v_{max} = A\omega = (0.10)(20) =$ **2.0 m/s**.
3. $E = \frac{1}{2}kA^2 = \frac{1}{2}(200)(0.10)^2 =$ **1.0 J**.
4. Check: $\frac{1}{2}m\,v_{max}^2 = \frac{1}{2}(0.50)(2.0)^2 = 1.0$ J ✓
Answer: T = 0.31 s, v_{max} = 2.0 m/s, E = 1.0 J

3. $T \propto \sqrt{m}$, so it **doubles**.

4. $\Delta y = mg/k =$ **4.9 cm**; $T = 2\pi\sqrt{(1.5/300)} =$ **0.44 s**.

5. Period **unchanged**; v_{max} **×2**; energy **×4**.

6. $k_{eff} = k/2$, so T increases by a factor of **√2**.

7. At the **endpoints** ($x = \pm A$), where the restoring force is greatest and the speed is zero.

8. $v_{max} = A\omega =$ **0.20 m/s**; $a_{max} = A\omega^2 =$ **1.0 m/s²**.