Vectors, Dot & Cross Products

Math toolkit · 7 questions · step-by-step answers at the end

Name: Date: Score: / 7

USEFUL EQUATIONS

$|\mathbf{A}| = \sqrt{(A_x^2 + A_y^2 + A_z^2)}$ $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$ $|\mathbf{A}\times\mathbf{B}| = AB\sin\theta$

$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{K}},\ \hat{\mathbf{j}}\times\hat{\mathbf{K}} = \hat{\mathbf{I}},\ \hat{\mathbf{K}}\times\hat{\mathbf{I}} = \hat{\mathbf{j}}$ $W = \mathbf{F}\cdot\mathbf{D},\ \mathbf{T} = \mathbf{R}\times\mathbf{F},\ \mathbf{F} = q\mathbf{V}\times\mathbf{B}$

Questions

Show your working in the space under each question.

1. Find the angle between $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{B} = \hat{\mathbf{i}}$.

2. A hiker walks 3.0 km east and then 4.0 km at 60° north of east. Find the magnitude and direction of her total displacement.

3. Two forces of 6 N and 8 N act on an object. What are the largest and smallest possible net forces?

4. Find the value of b that makes $\mathbf{A} = 2\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ perpendicular to $\mathbf{B} = 6\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$.

5. To cross the river above in the *shortest distance*, which way should the boat point?

6. A position vector $\mathbf{r} = 2\hat{\mathbf{i}}$ m and a force $\mathbf{F} = 5\hat{\mathbf{j}}$ N act on a wrench. Find the torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$.

7. What is $|\mathbf{A} \times \mathbf{B}|$ if $|\mathbf{A}| = 4$, $|\mathbf{B}| = 5$ and $\mathbf{A} \cdot \mathbf{B} = 10$?

Step-by-step answers

Vectors, Dot & Cross Products

1. $\mathbf{A}\cdot\mathbf{B} = 1$, $|\mathbf{A}| = \sqrt{2}$ and $|\mathbf{B}| = 1$, so $\cos\theta = 1/\sqrt{2}$ and $\theta = \mathbf{45°}$.

2. 1. First leg: (3.0, 0) km.
2. Second leg: (4.0 cos 60°, 4.0 sin 60°) = (2.0, 3.46) km.
3. Sum: (5.0, 3.46) km → magnitude $\sqrt{(25 + 12.0)} = \mathbf{6.1\ km}$.
4. Direction: $\tan\theta = 3.46/5.0 \rightarrow \theta = \mathbf{35°\ north\ of\ east}$.

Answer: 6.1 km at 35° N of E

3. Largest when parallel, **14 N**; smallest when opposite, **2 N**. At 90° apart the net force is 10 N.

4. Perpendicular means $\mathbf{A}\cdot\mathbf{B} = 0$: $12 - 4b = 0$, so $\mathbf{b = 3}$.

5. Angled upstream so that the current is cancelled: $\sin\theta = 3/4$, so $\theta \approx$ **49° upstream**. Its speed across is then $\sqrt{(16 - 9)} \approx 2.6$ m/s, so the crossing takes longer (≈ 30 s).

6. $\boldsymbol{\tau} = (2)(5)(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = \mathbf{10\hat{k}\ N{\cdot}m}$, which is counter-clockwise, out of the page.

7. $\cos\theta = 10/20 = 0.5$, so $\theta = 60°$ and $|\mathbf{A} \times \mathbf{B}| = 20 \sin 60° \approx \mathbf{17.3}$.